Multivariable Control System Design for Quadruple Tank Process using Quantitative Feedback Theory (QFT)

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***Abstract*— This paper focus on design of multivariable controller for Quadruple Tank Process, a two input two output system with large plant uncertainty using QFT methodology. In the present work, a new approach using Quantitative Feedback Theory (QFT) is formulated for design of a robust two degree of freedom controller for Quadruple Tank Process. The design is done in frequency domain. This paper presents a design method for a 2 x 2 multiple input multiple output system. The plant uncertainties are transformed into equivalent external disturbance sets, and the design problem becomes one of the external disturbance attenuation. The objective is to find compensator functions which guarantee that the system performance bounds are satisfied over the range of plant uncertainty. The methodology is successfully applied to design a two degree of freedom compensator Quadruple Tank Process.**

Kewords — **Quadruple Tank Process, Robust Control, QFT, MIMO, EDA.**

I. INTRODUCTION

Many practical applications are multi-input multi-output (MIMO) systems. MIMO system is composed of several interacting control loops and numbers of alternative configurations of control loops are very large. Interaction produces several undesirable effects. The design method proposed in this paper consists of design of multivariable controller to have desired response from the plant in the presence of plant uncertainties and external disturbances. The quadruple tank process is a multivariable laboratory process developed by K.H. Johansson[7]. This process consists of four interconnected water tanks and two pumps as shown in figure 1. Here objective to control the level in the lower two tank with two pump.

The Quantitative Feedback Theory (QFT) is a control method in frequency domain which uses the Nichols chart [1]. It is an engineering science devoted to the design problem with quantitative bounds on the plant parameters and quantitative tolerances on the acceptable closed-loop system response. The objective is to find compensator functions which guarantee that the system performance bounds are satisfied over the range of plant uncertainty.

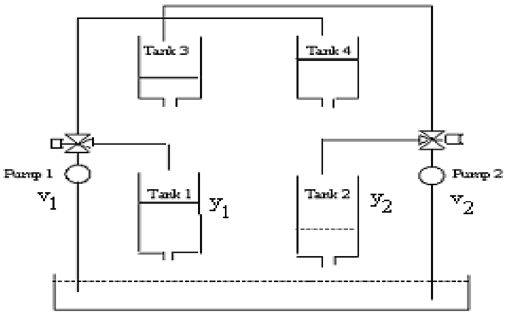

Figure 1: Quadruple tank system.

In this work, we propose a QFT based methodology to design a robust two degree of freedom MIMO compensator for quadruple tank process in the presence of parametric uncertainty. The plant uncertainties are transformed into equivalent external disturbance sets and thus the design problem becomes one of external disturbance attenuation (EDA). The method has been proven rigorous by means of Schauder's fixed point theorem [2]. The design technique is successfully applied to design a two degree of freedom compensator for quadruple tank process.

The present paper is organized as follows: Section II deals with the problem definition for a 2 x 2 MIMO design problem. Basic concepts are discussed in section III. Design equations used in the synthesis of controller for quadruple tank process is given in section IV. Section V deals with the design procedure to design controller and prefilter. In section VI, the methodology is applied to design a compensator for quadruple tank process. Conclusions are drawn in section VII.

II. A 2X2 MIMO DESIGN PROBLEM

The two degree of freedom controller structure used for the design of two input and two output quadruple tank process is shown in figure 2.

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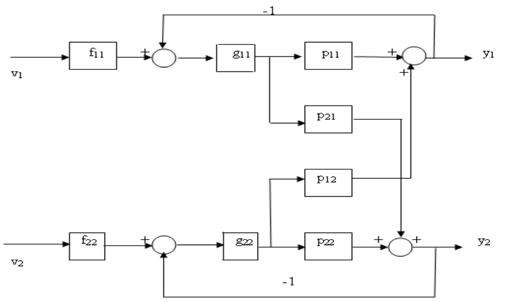

Figure 2: Controller Structure for 2 x 2 MIMO System.

P= [p_{ij} (s)] represents the set of all possible plant transfer function elements. If t(s) represents the closed loop transfer function matrix between Y(s) and U(s) and T(s) represents the overall closed loop transfer function matrix between Y(s) and R(s), then,

T (s) = t (s) x F (s)

Where,

$$T=\begin{bmatrix}T_{11} & T_{12}\\ T_{21} & T_{22}\end{bmatrix},\ t=\begin{bmatrix}t_{11} & t_{12}\\ t_{21} & t_{22}\end{bmatrix} and\ F=\begin{bmatrix}f_{11} & 0\\ 0 & f_{22}\end{bmatrix}$$

F(*s*) is the forward filter designed to achieve the desired tracking properties for *T*(*s*). The controller placed inside the feedback loop has a diagonal structure given by,

$$G=\begin{bmatrix}g_{11} & o\\ 0 & g_{22}\end{bmatrix}$$

Both *G*(*s*) and *F*(*s*) are rational and strictly proper linear time invariant matrices.

III. BASIC CONCEPTS

Consider a two degree-of-freedom controller structure as illustrated in figure 3, where P represents the set of transfer function which describe the region of plant uncertainty, G is the compensator and F is an input pre-filter transfer function. The system has two inputs: R the desired input signal to be tracked and D an external disturbance input signal which is to be attenuated to have minimal effect on output Y.

The controller and prefilter are to be synthesized in order to meet robust stability and closed loop specifications. Compensator is designed so that the variations of output due to the uncertainty in the plant are within allowable tolerance and effects of disturbances are small. The compensator G is designed via loop shaping process. The

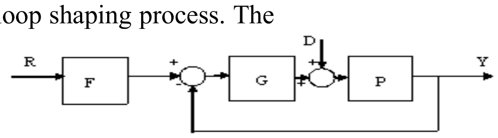

Figure 3: Two degree of freedom controller.

loop shaping is done to generate a controller G(s), such that the open loop transfer function L(s) = GP satisfies certain specifications such as Nyquist stability, gain-margin, phase-margin etc. Once the compensator is designed the closed loop Bode plots will be as close together as the tracking specifications but they will not usually fall within the tracking specifications. A prefilter F is designed which brings the frequency response of the closed loop system within the tracking specifications. In figure 4, *P* is the set (due to uncertainty) of the n x n plant transfer function matrix P = [pij(s)]. The elements of the n x n compensator G = [g_{ij}(s)] and the n x n pre-filter F = [f_{ij}(s)] are to be chosen practically (each with an excess of poles over zero). They must ensure that the closed loop response falls inside the desired bounds over all P Є *P*. An alternative method presented in [2] involved the replacement of the plant uncertainty entirely by an equivalent disturbance set and normal fixed, certain plant.

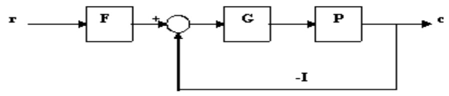

Figure 4: A Canonical structure.

Consider a two- degree of freedom, n x n input-output feedback system, as shown in figure 4. Let P_0, P be the n x n nominal and perturbed plant transfer matrix, respectively. L_0 = P_0G is the nominal loop transmission matrix T_0 = [t_{ij0}], T = [t_{ij}] are the nominal and perturbed system transfer matrix, respectively. The n x 1 output matrix c can be expressed as,

$$c=PG(Fr-c) \quad (3)$$

Where r an n x 1 input matrix, F is an n × n fixed pre-filter matrix of transfer function.

$$(I+PG)c=PGFr \quad (4)$$

P must be nonsingular matrix for the system to be controllable, so

$$(P^{-1}+G)c=GF$$
$$(P_0P^{-1}+P_0G)T=P_0GF$$
$$(P_0P^{-1}+L_0)T=L_0F \quad (5)$$

Define

$$V=I-P_0P^{-1}, \quad (6)$$

$$so\quad P_0P^{-1}=I-V \quad (7)$$

$$(I+L_0)=L_0F+VT \quad (8)$$

The nominal system transfer matrix

$$T_0=(I+L_0)^{-1}L_0F \quad (9)$$

Giving

$$(I+L_0)(T-T_{0)}=VT$$
$$Let\quad \Delta T=[\Delta t_{ij}]=T-T_0$$
$$(I+L_0)\Delta T=VT$$

Since $(I+L_0)^{-1}$ exist

Therefore,

$$\Delta T=(I+L_0)^{-1}VT. \quad (10)$$

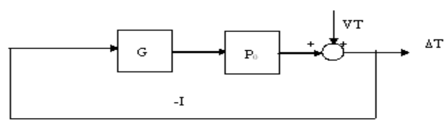

Figure 5: Disturbance attenuation only.

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Equation 10 is implement in figure 5 with the equivalent disturbance matrix D_e = VT. The uncertainty in P is transformed into the equivalent disturbance term VT; hence, it is called the "equivalent disturbance method". As T roams over its acceptable set, D_e describes a corresponding set.

IV. DESIGN EQUATIONS

From equation 10,
If L_0 is taken as diagonal then

$$\Delta t_{ij} = \frac{v_{ij}t_{ij} + \sum_{k\neq i}^{n} v_{ik}t_{kj}}{1+l_{ii0}}, \; j = 1,2,3........n \quad (11)$$

From equations 11 the design equations are derived as follows, For i = j

$$\frac{1}{1+l_{ii0}} = \frac{\frac{\Delta t_{ii}}{t_{ii}}}{(v_{ii} + \sum_{k\neq i}^{n} \frac{v_{ik}t_{ki}}{t_{ii}})} \quad (12)$$

$$\left|\frac{1}{1+l_{ii0}}\right|_{dB} = \left|\frac{\Delta t_i i}{t_{ii}}\right|_{dB} - \left|v_{ii} + \sum_{k\neq i}^{n} \frac{v_{ik}t_{ki}}{t_{ii}}\right|_{dB}$$

$$\left|\frac{1}{1+l_{ii0}}\right|_{dB} = |t_{iiv}|_{dB} - |p_{iiv}|_{dB}$$

Substitute

$$t_{iiv} = \frac{\Delta t_{ij}}{t_{ii}}$$

$$p_{iiv} = v_{ii} + \sum_{k\neq i}^{n} \frac{v_{ik}t_{ki}}{t_{ii}}$$

$$\left|\frac{1}{1+l_{ii0}}\right|_{dB} = |t_{iiv}|_{dB} - |p_{iiv}|_{dB} \quad (13)$$

Similarly from equation 11, for i ≠ j

$$\left|\frac{1}{1+l_{ii0}}\right|_{dB} = |t_{ijv}|_{dB} - |p_{ijv}|_{dB} \quad (14)$$

where,

$$t_{iiv} = \frac{\Delta t_{ij}}{t_{ii}}$$

$$p_{ijv} = v_{ij} + \sum_{k\neq i}^{n} \frac{v_{ik}t_{ki}}{t_{ji}}$$

The normal loop transmission function l_{iio} should be designed to satisfy

$$\left|\frac{1}{1+l_{ii0}}\right|_{dB} \leq \min\left(|t_{iiv}|_{dB} - |p_{iiv}|_{dB\max}, \; |t_{ijv}|_{dB} - |p_{ijv}|_{dB\max} \right) \quad (15)$$

Let $h_{ij} = \frac{1}{l_{ii0}}$

$$\left|\frac{1}{1+h_{ii0}}\right|_{dB} = \left|\frac{1}{1+l_{ii0}}\right|_{dB} \quad (16)$$

The Nichols chart with its loci of constant $|L/(1+L)|$ etc an be for finding the bounds on l_{iio} to guarantee the desired performance. The bounds of $l_{iio}(s)$ are calculated by using equation 15 and 16. Loop shaping is done such as at each frequency l_{ii0} must lie on or above the specified frequency bound.

V. CONTROLLER SYNTHESIS PROCEDURE

The design procedure to design the prefilter F(s) and the compensator G(s), to achieve specific robust design for given region of plant parameter uncertainty is as follows:

1. Translate of time domain specifications into frequency domain specifications.
2. Stability bound generation.
3. Conversion of tracking specifications into disturbance.
4. Disturbance bound generation.
5. Grouping of bounds.
6. Intersection of bounds.
7. Design of controller G(s).
8. Design prefilter F(s) for tracking of reference.

VI. QUADRUPLE TANK APPLICATION

The quadruple tank system is as shown in figure 1. Here objective to control the level in the lower two tank with two pump. The process inputs are v_1 and v_2 (input voltages to the pump) and the outputs are y_1 and y_2 (voltages from level measurement device). The positions of the valves determine the location of multivariable zero for the linearized model. The zero can be put in either the left or the right plane. Thus system may have minimum phase characteristics or non minimum phase characteristics. Here we will consider only minimum phase system [7].

Let,

$$P(s) = \begin{bmatrix} \frac{2.6}{1+62s} & \frac{1.5}{(1+25s)(1+62s)} \\ \frac{1.4}{(1+30s)(1+90s)} & \frac{2.8}{(1+90s)} \end{bmatrix}$$

The interval plants with uncertainty are

$$p_{11} = \frac{k_1}{1+a_1 s}, p_{12} = \frac{k_2}{(1+23s)(1+a_2 s)}$$

$$p_{21} = \frac{k_3}{(1+30)(1+a_3 s)}, p_{22} = \frac{k_4}{(1+a_4 s)}$$

where,

$k_1 \in [2.4,\ 2.8]$; $k_2 \in [1.4, 1.6]$;
$k_3 \in [1.3,\ 1.5]$; $k_4 \in [2.6, 2.8]$
$a_1 \in [58,\ 66]$; $a_2 \in [58, 66]$;
$a_3 \in [85,\ 95]$; $a_4 \in [85, 95]$;

The performance specification for tank 1 and tank 2 are: 1. Tracking specifications: 16 sec. ≤ t_{11} (t) ≤ 27 sec. and 10% overshoot. 2. Stability specifications: GM ≥ 6 dB and PM ≥ 40^0. 3. Interaction ≤ 10%.

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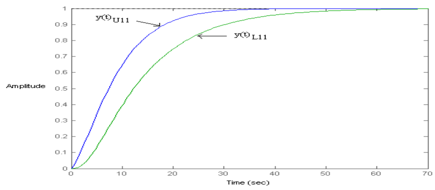

Figure 6: System time domain tracking performance specifications for tank-1.

The time domain specifications are as indicated in figure 6. The time domain specifications are translated into the frequency domain. The frequency specifications are defined by an upper bound B_U and lower bound B_L as shown in figure 7. The desired control ratios corresponding to upper and lower bound are synthesized as the dB difference between upper bound (B_U) and lower bound (B_L) increases with frequency in high frequency range. The desired spread between B_U and B_L is achieved

by augmenting B_U with zero as close as possible to origin without significantly affecting the time response. The spread can be further increased by augmenting B_L

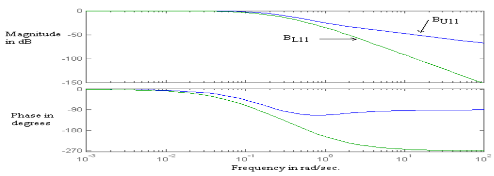

Figure 7: Frequency domain performance specifications (Bode plot) for tank-1.

with negative real pole which is as close to origin as possible but far enough away not to significantly affect the time response.

The equivalent frequency domain transfer functions of upper and lower tracking bounds for tank-1 are as follows:

$$B_{u11} = \frac{0.04s + 0.04}{s^2 + 0.4s + 0.04}$$

$$B_{l11} = \frac{0.02457}{s^3 + 1.528s^2 + 0.3757s + 0.02457}$$

The bounds are generated at different frequencies for given stability specification. The disturbance specifications are calculated by using equation 15 and 16. All the bounds are plotted for given stability and disturbance specifications. The most dominating bounds are selected for controller design at each frequency. The bounds on l_{110} and nominal loop shaping of l_{110} for tank 1 are as shown in figure 8.

The corresponding compensator transfer function for tank 1 is as follows:

$$g_{11} = \frac{21.20(s + 0.06119)}{s(s + 0.7758)(s + 8.553)}$$

The prefilter for tank 1 is as shown in figure 9. The corresponding prefilter for tank 1 are as follows:

$$f_{11} = \frac{0.067}{(s + 0.067)}$$

Similar procedure is carried out for tank 2. The bounds on l_{220} and nominal loop shaping of l_{220} for tank 2 are as shown in figure 9. The corresponding compensator transfer function and the prefilter for tank 2 are as follows:

$$g_{22} = \frac{54.63(s + 0.06785)}{s(s + 2.135)(s + 4.585)}$$

The prefilter for tank 2 is

$$f_{22} = \frac{0.072}{(s + 0.072)}$$

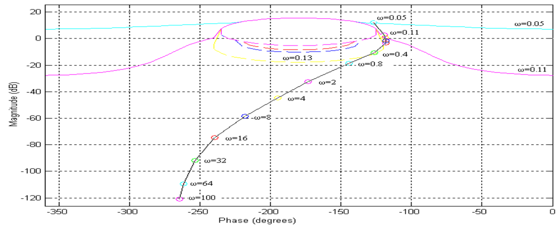

Figure 8: The bounds on l_{110} and nominal loop shaping of l_{110} for tank 1.

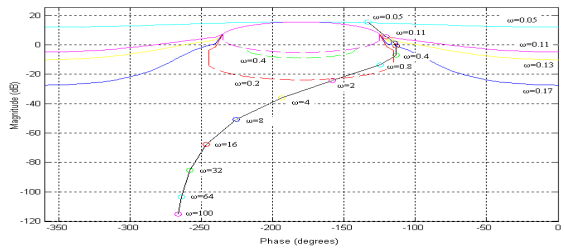

Figure 9: The bounds on l_{220} and nominal loop shaping of l_{220} for tank 2.

The SIMULINK block diagram for MIMO system is as shown in figure 10.

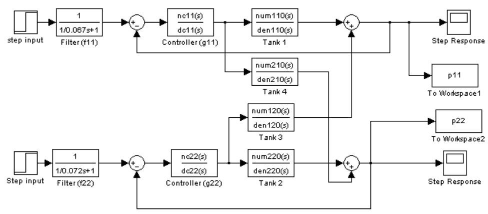

Figure 10: The SIMULINK block diagram for MIMO system.

The closed loop responses of entire plant with step input at v_1 with the designed compensator is within the specified upper and lower bounds as shown in figure 11. Same is true for step input at v_2 as shown in figure 12.

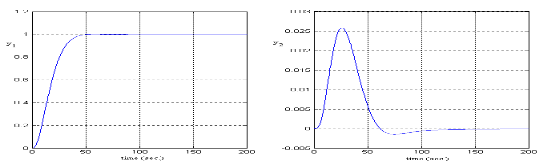

Figure 11: Closed loop responses y_1 and y_2 with unit step input at v_1.

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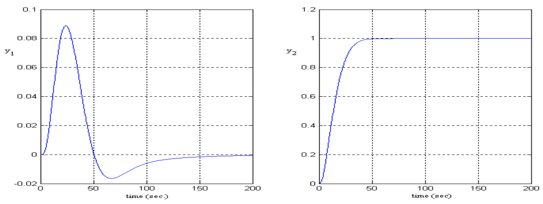

Figure12: Closed loop responses y_1 and y_2 with unit step input at v_2.

The closed loop response of entire plant with step input at v_1 and v_2 with the designed compensator is within the specified upper and lower bounds as shown in figure 14.

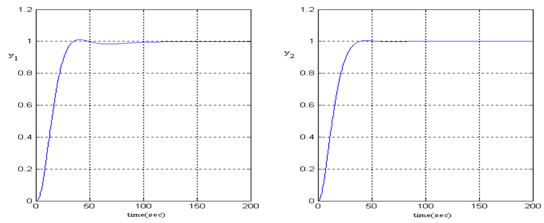

Figure 14: Closed loop responses y_1 and y_2 with unit step input at v_1 and v_2.

VII CONCLUSION

In the proposed method, the tracking specifications are converted into disturbance specifications, because of which the design for MIMO system becomes simpler. The EDA method is successfully applied to the Quadruple Tank application. The result shows that the desired specifications are achieved by both the controlled variables. The result also shows that the interactions between the loops are within specified limits.

Here, the EDA methods based on QFT methodology shows power of QFT. The result proves that QFT methods can be used to handle large uncertainties.

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